Pergamon

Int. J. Solids Structures Vol. 31, No. 4, pp. 457–467, 1994

0020–7683/94 $6.00 + .00

EQUIVALENT CONTINUUM MODELS OF LARGE PLATELIKE LATTICE STRUCTURES

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(*Received* 10 *March* 1993; *in revised form* 16 *August* 1993)

Abstract—A new continuum method based on the concept of energy equivalence is introduced to represent a large platelike periodic lattice structure as an equivalent continuum plate. The procedure for developing an equivalent continuum plate involves the use of existing well-defined finite-element matrices for easy calculation of strain and kinetic energies contained in a representative lattice cell of the lattice plate. The strain and kinetic energies contained in the representative lattice cell of lattice plate and the finite-element of equivalent continuum plate are expressed in terms of continuum degrees-of freedom introduced in this paper with corresponding reduced stiffness and mass matrices. The structural properties of the equivalent continuum plate are then obtained by simply equating the reduced stiffness and mass matrices for the equivalent continuum plate to those for the lattice plate. Free vibration analyses for the continuum plates are conducted to evaluate the continuum method proposed in this paper. Numerical tests show that the present continuum method gives very reliable structural and dynamic properties compared with other well-recognized continuum methods.

1. INTRODUCTION

Large space structures (LSS) have been proposed with dimensions of many kilometers. Such structures will be designed to be composed of identically constructed cell units which are connected end-to-end to form a spatially periodic array (Card, 1978). Structural and dynamic characteristics of the LSS must be predicted accurately during the initial design phase, since they cannot be tested full scale in their operational environments prior to flight. Conventional finite-element analysis of the LSS may require a significant amount of storage capacity and computing time to obtain reliable solutions because of its high structural flexibility and large size, especially in the dynamic analysis. Thus special techniques to cope with the very large number of elements and nodes within such large lattice structures are desired.

Alternative methods have been developed for simplified structural modeling of the lattice structures composed of many repeating cells (see Noor *et al.*, 1978). Of these methods, the simplification of a period lattice structure by an equivalent continuum model, often named as continuum modeling, is known to provide a very promising and practical solution method for overall vibration modes and structural responses. The key to continuum modeling involves the determination of appropriate relationships between the geometric and material properties of lattice structure and those of the equivalent continuum model. Because there can be numerous concepts and ways of determining the appropriate relationships between lattice and continuum domains, the continuum modeling itself may not be unique and can fall in one of several distinct categories. In order to avoid an excessively long bibliography, the reader is referred to the reference works by Power and Schmidt (1971), Noor *et al.* (1978), Nayfeh and Hefzy (1981) and Sun *et al.* (1988).

The continuum models based on the concept of energy equivalence have shown to give satisfactory results when the wavelength of a vibration mode spans many repeating cells of a lattice structure (see Noor *et al.*, 1978). Here, energy equivalence means that the lattice structure and the equivalent continuum model must contain equal kinetic and strain energies when both domains are subjected to the same displacement and velocity fields. The accuracy and convenience of the energy equivalence concept generally depend on the way the strain and kinetic energies, contained in a finite domain of lattice structure or equivalent continuum model, are calculated. Recently Lee (1990) proposed a new energy equivalence method and applied it to the large beamlike lattice structures. The new method involves the idea of using conventional finite-element matrices in calculating the strain and kinetic energies stored in a representative lattice cell, which is different from the continuum method by Noor (1978) in which he used complicated kinematics and assumptions to replace the displacements and strains in a lattice element by the Taylor series expansions of the displacement and strain components in the coordinate directions, prior to energy integrations.

The objectives of the present paper are: (1) to propose a simple and rational method for developing continuum models of large platelike lattice structures (simply, lattice plates), as an extension of the continuum method developed previously for the beamlike lattice structures by the author (Lee, 1990), and (2) to evaluate the accuracy and validity of the continuum method developed herein by means of numerical tests.

2. DEVELOPMENT OF EQUIVALENT DYNAMIC CONTINUUM MODELS

2.1. *General procedure of continuum modeling*

This paper develops a procedure for formulating an equivalent homogeneous anisotropic continuum plate (simply, continuum plate) representation of a large lattice plate composed of many repeating cells. For the continuum modeling, three basic assumptions on the lattice plate are made: (1) it behaves grossly as a continuum plate especially in lower mode vibrations, (2) the ratio of inplane dimensions to thickness is very high (more than 20 in general), and (3) the deflection is small compared to the thickness. Then the lattice plate can be modeled as a classical thin plate (Whitney, 1987), neglecting the effects of transverse shear deformation and rotatory inertia. However, it is well known that transverse shear deformation is frequently important when plate theory is used to model systems such as truss-type lattice structures (Langhaar, 1962). Also, when a platelike lattice structure is not symmetric with respect to its midplane, couplings between extension, transverse shear and bending deformations may be significant. To improve the continuum models of lattice plate, therefore, the continuum modeling procedure introduced herein includes the shear and rotatory effects as well as the couplings between three basic deformations, which will be important especially for the dynamic behavior of truss-type lattice structures.

The continuum modeling based on energy equivalence needs the calculation of the strain and kinetic energies stored in a representative lattice cell. It is well known that stiffness and mass matrices for a finite-element can be derived directly from the strain and kinetic energies calculated after making a reasonable appropriate hypothesis for the displacement fields within the finite-element. The finite-element matrices for a truss (axial bar) or beam element are readily available from many text books. Since the convenience of energy equivalence concept mostly depends on how the energies contained in a finite domain are calculated easily, it seems to be easy and rational to utilize the finite-element matrices in calculating the strain and kinetic energies of a representative lattice cell. Briefly the modeling procedure consists of the following steps:

(1) a representative lattice cell is isolated from a periodic lattice plate (Fig. 1);

(2) appropriate continuum degrees-of-freedom (DOFs) are introduced to condense the stiffness and mass matrices which can be derived from the strain and kinetic energy expressions;

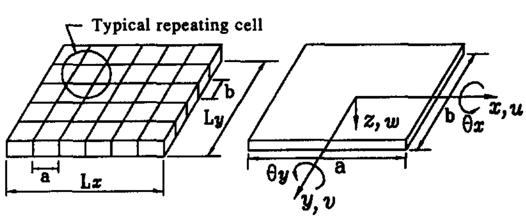

Fig. 1. Equivalent continuum plate element for a typical repeating cell.

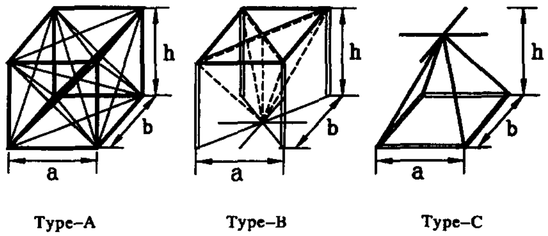

Fig. 2. Typical repeating cells considered in present study.

(3) transformation matrices which relate the nodal DOFs for each lattice node to the continuum DOFs, are formulated;

(4) the strain and kinetic energies stored in a representative lattice cell are obtained in terms of continuum DOFs by utilizing well-defined existing finite-element matrices for a truss or beam element. Reduced stiffness and mass matrices for a representative lattice cell are then derived from these energy expressions;

(5) reduced stiffness and mass matrices for a finite-element of continuum plate are also derived by expressing the strain and kinetic energies stored in a finite-element of continuum plate as the functions of continuum DOFs. The reduced matrices will contain all equivalent structural properties of the continuum plate to be determined;

(6) the equivalent structural properties of a continuum plate are finally obtained by simply equating the reduced stiffness and mass matrices for the representative lattice cell to those for the finite-element of continuum plate.

2.2. *Reduced stiffness and mass matrices for a representative lattice cell*

Three rectangular lattice plate models having different types of repeating cells, as shown in Fig. 2, will be transformed into the equivalent rectangular continuum plates. The repeating cells are composed of several different lattice elements: top and bottom surfaces bars, vertical bars and diagonal bars. Generally the repeating cells are so constructed that all joints are located on one of top, bottom and middle surfaces. Figure 3 shows the three distinct surfaces on which joints are located. As in conventional finite-element analysis, the joints will be considered as nodal points (or nodes) at which nodal DOFs are defined. In Figs 1 and 3, u and v are the inplane displacements in the x- and y-directions, w the transverse deflection in the z-direction, and θ_x and θ_y are the rotations about the x- and y-axes, respectively.

The nodal DOFs at each node can be combined to be expressed in the form of displacement vectors as indicated in Fig. 3. The nodal displacements vector at the i-th node of (x_i, y_i, z_i) on the top or bottom surface is defined by:

$$\{\delta_i\} = \{u_i, v_i, w_i\}^T. \tag{1}$$

Three rotational DOFs can be readily added to $\{\delta_i\}$ in the case of nonhinged joints. The nodal displacement vector at $(x_i, y_i, 0)$ in midsurface is now defined by:

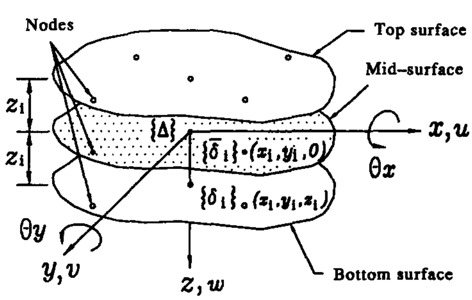

Fig. 3. Nodal DOFs and sign convention for a repeating cell.

$$\{\bar{\delta}_i\} = \{\bar{u}_i, \bar{v}_i, \bar{w}_i, \bar{\theta}_{xi}, \bar{\theta}_{yi}\}^T. \tag{2}$$

Where two rotational DOFs are included to account for the inplane deformations due to bending, which are linearly proportional to the distance from the midsurface under the assumption of small deformation. Then the approximated relation between $\{\delta_i\}$ and $\{\bar{\delta}_i\}$ is found as follows:

$$\{\delta_i\} = \begin{bmatrix} 1 & 0 & 0 & 0 & z_i \\ 0 & 1 & 0 & -z_i & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \{\bar{\delta}_i\} = [A_i]\{\bar{\delta}_i\}. \tag{3}$$

We now introduce the continuum DOFs at the center of midsurface (0, 0, 0), in the vector form as:

$$\{\Delta\} = \begin{Bmatrix} \Delta^R \\ \Delta^E \end{Bmatrix}, \tag{4}$$

where

$$\{\Delta^R\} = \{u_o, v_o, w_o, \theta_{xo}, \theta_{yo}\}^T \tag{5}$$

$$\{\Delta^E\} = \{\varepsilon_{xo}, \varepsilon_{yo}, \gamma_{xyo}, \gamma_{xzo}, \gamma_{yzo}, \varkappa_{xo}, \varkappa_{yo}, \varkappa_{xyo}\}^T. \tag{6}$$

The vector $\{\Delta^R\}$ represents the rigid-body displacements vector at the center of the midsurface, and the vector $\{\Delta^E\}$ represents the strains and curvatures vector due to elastic deformations. The vector $\{\Delta^E\}$ is obviously responsible for the strain energy storage, while the vector $\{\Delta^R\}$ is mainly responsible for the kinetic energy storage. Using Taylor series expansion, and neglecting higher order terms under the assumption of small deformation, $\{\bar{\delta}_i\}$ can be also related to $\{\Delta\}$, approximately in the form of:

$$\{\bar{\delta}_i\} = [B_i]\{\Delta\} = [B_i^R B_i^E]\{\Delta\}. \tag{7}$$

The transformation matrix $[B_i]$ is given in Table 1. Combining eqns (3) and (7), one can represent the nodal displacements as the sum of two parts: the part of rigid-body displacements and the part of elastic deformations. That is:

$$\{\delta_i\} = \{\delta_i^R\} + \{\delta_i^E\}, \tag{8}$$

where

$$\{\delta_i^R\} = [A_i][B_i^R]\{\Delta^R\} = [R_i^R]\{\Delta^R\}, \tag{9}$$

$$\{\delta_i^E\} = [A_i][B_i^E]\{\Delta^E\} = [R_i^E]\{\Delta^E\}. \tag{10}$$

Thus the rigid-body displacements and the elastic deformations at each node can be

Table 1. Transformation matrix: $[B_i] = [B_i^R B_i^E]$

$[B_i^R]$	$[B_i^E]$
$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & y_i & -x_i \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} x_i & 0 & y_i/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & y_i & y_i/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_i & y_i & -x_i^2/2 & -y_i^2/2 & -x_iy_i/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -y_i & -x_i/2 \\ 0 & 0 & 0 & 0 & 0 & x_i & 0 & y_i/2 \end{bmatrix}$

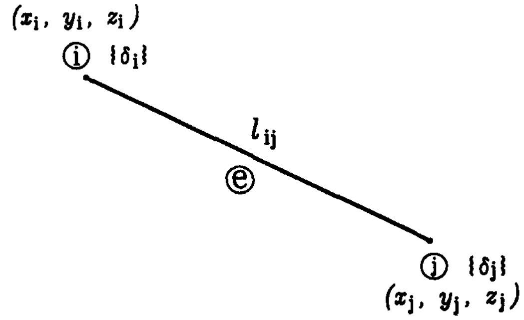

Fig. 4. A typical lattice element.

represented as the functions of continuum DOFs through eqns (8)–(10). Figure 4 shows the e-th lattice element between two nodes, i and j. If we know the coordinates of the two nodes, the length and direction cosines of the lattice element can be calculated to construct a coordinate transformation matrix $[S_e]$ which transforms the local coordinates attached to the lattice element into the global coordinates. The finite-element stiffness and mass matrices for the lattice element with respect to global coordinates are then obtained from:

$$[\bar{k}_e] = [S_e]^T[k_e][S_e], \tag{11}$$

$$[\bar{m}_e] = [S_e]^T[m_e][S_e], \tag{12}$$

where $[k_e]$ and $[m_e]$ are the finite-element matrices with respect to local coordinates. Using the finite-element matrices of eqns (11) and (12), the strain and kinetic energies stored in the e-th lattice element are obtained from:

$$V_e = \tfrac{1}{2}\{d_e^E\}^T[\bar{k}_e]\{d_e^E\}, \tag{13}$$

$$T_e = \tfrac{1}{2}\{\dot{d}_e^R\}^T[\bar{m}_e]\{\dot{d}_e^R\}, \tag{14}$$

where

$$\{d_e^E\} = \begin{Bmatrix} \delta_i^E \\ \delta_j^E \end{Bmatrix} = \begin{bmatrix} R_i^E \\ R_j^E \end{bmatrix}\{\Delta^E\} = [R_e^E]\{\Delta^E\}, \tag{15}$$

$$\{d_e^R\} = \begin{Bmatrix} \delta_i^R \\ \delta_j^R \end{Bmatrix} = \begin{bmatrix} R_i^R \\ R_j^R \end{bmatrix}\{\Delta^R\} = [R_e^R]\{\Delta^R\}. \tag{16}$$

The element energies of eqns (13) and (14) can be expressed as the functions of continuum DOFs by substituting eqns (15) and (16) into them. Summing all element energies, total energies stored in a representative lattice cell are obtained from:

$$V_L = \sum_e V_e = \tfrac{1}{2}\{\Delta^E\}^T[K_L]\{\Delta^E\}, \tag{17}$$

$$T_L = \sum_e T_e = \tfrac{1}{2}\{\dot{\Delta}^R\}^T[M_L]\{\dot{\Delta}^R\}, \tag{18}$$

where $[K_L]$ and $[M_L]$ are the reduced stiffness and mass matrices for a representative lattice cell defined by:

$$[K_L] = \sum_e [R_e^E]^T[\bar{k}_e][R_e^E], \tag{19}$$

$$[M_L] = \sum_e [R_e^R]^T[\bar{m}_e][R_e^R]. \tag{20}$$

Judging from the definition of continuum DOFs introduced in eqn (4), $[K_L]$ and $[M_L]$ are found to be 8×8 and 5×5 symmetric matrices, respectively.

2.3. *Reduced stiffness and mass matrices for a finite-element of continuum plate*

A representative lattice cell of the periodic lattice plate is so transformed into a rectangular finite-element of continuum plate that both lattice plate and continuum plate are structurally and dynamically equivalent to each other. Assuming the midsurface of a continuum finite-element coincides with the midsurface of the corresponding representative lattice cell, the same nodal displacements vector $\{\delta_i\}$ as defined in eqn (2) for the representative lattice cell is again introduced at each node of continuum finite-element. The nine-noded finite-element, as an example, is shown in Fig. 5. Based on the shear deformation theory of anisotropic plate (Chia, 1980), the strain energy within the continuum finite-element of area A_e can be obtained from:

$$V_C = \frac{1}{2}\int_{A_e} \begin{Bmatrix} N \\ Q \\ M \end{Bmatrix}^T \begin{Bmatrix} \varepsilon \\ \gamma \\ \varkappa \end{Bmatrix} \mathrm{d}A, \tag{21}$$

where $\{N\} = \{N_x, N_y, N_{xy}\}^T$ are the resultant membrane forces, $\{Q\} = \{Q_x, Q_y\}^T$ the resultant transverse shear forces, and $\{M\} = \{M_x, M_y, M_{xy}\}^T$ the resultant moments, all per unit length. Also the inplane strains $\{\varepsilon\}$, the transverse shear strains $\{\gamma\}$, and the bending and twisting curvatures $\{\varkappa\}$ can be expressed in terms of displacement derivatives as follows:

$$\{\varepsilon\} = \left\{\frac{\partial u}{\partial x},\ \frac{\partial v}{\partial y},\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right\}^T, \tag{22}$$

$$\{\gamma\} = \left\{\frac{\partial w}{\partial x} + \theta_y,\ \frac{\partial w}{\partial y} - \theta_x\right\}^T, \tag{23}$$

$$\{\varkappa\} = \left\{\frac{\partial \theta}{\partial x},\ -\frac{\partial \theta_x}{\partial y},\ \frac{\partial \theta_y}{\partial y} - \frac{\partial \theta_x}{\partial x}\right\}^T. \tag{24}$$

The basic constitutive relations of the anisotropic plate are symbolically given as:

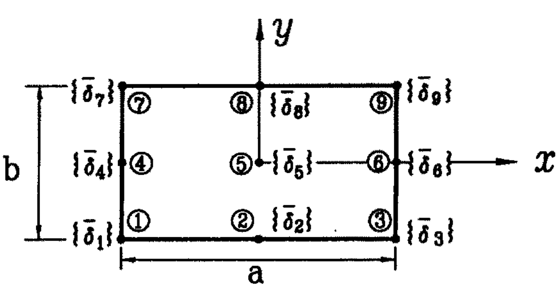

Fig. 5. Nine-noded finite-element for the homogeneous anisotropic plate.

$$\begin{Bmatrix} N \\ Q \\ M \end{Bmatrix} = \begin{bmatrix} A & F & B \\ F^T & S & H \\ B^T & H^T & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \gamma \\ \varkappa \end{Bmatrix} = [E] \begin{Bmatrix} \varepsilon \\ \gamma \\ \varkappa \end{Bmatrix}, \tag{25}$$

where symmetric matrices $[A]$, $[S]$ and $[D]$ are the membrane, transverse shear and bending rigidity symmetric matrices, respectively. The matrices $[F]$, $[B]$, and $[H]$ are the extension-transverse shear, extension-bending, and transverse shear-bending coupling rigidity matrices (Whitney, 1987). As mentioned earlier, the transverse shear and coupling effects may be important especially for the dynamic behavior of truss-type lattice structures. The displacements and rotations within a finite-element can be described in terms of the nodal displacements by use of interpolation through the shape functions $N_i(x, y)$, as follows:

$$(u, v, w, \theta_x, \theta_y) = \sum_{i=1}^{n} N_i(\bar{u}_i, \bar{v}_i, \bar{w}_i, \bar{\theta}_{xi}, \bar{\theta}_{yi}), \tag{26}$$

where n is the number of nodes within a finite-element. By substituting the elastic deformations part of eqn (26) into eqns (22)–(24), and using the relations of eqns (2) and (7), the strains and curvatures vector is rewritten in the form:

$$\begin{Bmatrix} \varepsilon \\ \gamma \\ \varkappa \end{Bmatrix} = \sum_{i=1}^{n} \begin{bmatrix} G_{mi} & 0 \\ 0 & G_{si} \\ 0 & G_{bi} \end{bmatrix} [B_i^E]\{\Delta^E\} = [\Omega]\{\Delta^E\}. \tag{27}$$

Defining $\alpha_i = \partial N_i/\partial x$ and $\beta_i = \partial N_i/\partial y$, the submatrices in eqn (27) are derived as follows:

$$[G_{mi}] = \begin{bmatrix} \alpha_i & 0 \\ 0 & \beta_i \\ \beta_i & \alpha_i \end{bmatrix}, \quad [G_{si}] = \begin{bmatrix} \alpha_i & 0 & N_i \\ \beta_i & N_i & 0 \end{bmatrix}, \quad [G_{bi}] = \begin{bmatrix} 0 & 0 & \alpha_i \\ 0 & \beta_i & 0 \\ 0 & \alpha_i & \beta_i \end{bmatrix}. \tag{28}$$

The eqns (25) and (27) are substituted into eqn (21) to express the strain energy as:

$$V_C = \tfrac{1}{2}\{\Delta^E\}^T[K_C]\{\Delta^E\}, \tag{29}$$

where $[K_C]$, the reduced stiffness matrix for a finite-element of continuum plate is given as:

$$[K_C] = \int_{A_e} [\Omega]^T[E][\Omega]\,\mathrm{d}A. \tag{30}$$

The matrix $[\Omega]$ is mainly dependent on the shape functions N_i and the coordinates (x_i, y_i, z_i) of nodal points within a finite-element. To compute $[\Omega]$, five different rectangular finite-elements are analytically tested. They are four-noded, nine-noded, and 16-noded Lagrange elements, and eight-noded and 12-noded serendipity elements (Huang, 1989). It is quite interesting to conclude that the matrix $[\Omega]$ always becomes the identity matrix for all finite-elements, except for the four-noded Lagrange element for which $[\Omega]$ is also very close to the identity matrix. This may be one of the important advantages of introducing the continuum DOFs defined in eqn (4). Hence, the reduced stiffness matrix for a continuum plate can be much simplified as follows:

$$[K_C] = A_e[E]_{eq}. \tag{31}$$

Now, the kinetic energy within a finite-element of continuum plate is given as:

$$T_C = \frac{1}{2}\int_{A_e} \{\delta^R\}^T[m]\{\delta^R\}\,\mathrm{d}A, \tag{32}$$

where $\{\delta^R\}$ is the rigid-body displacements part of $\{u, v, w, \theta_x, \theta_y\}^T$, and $[m]$ is the generalized inertia matrix which includes all coupling inertias as follows:

$$[m] = [m_{ij}] \quad (i,j = 1, \ldots, 5). \tag{33}$$

In general, $m_{11} = m_{22} = m_{33} = m$ are recognized as the mass per unit area, $m_{15} = -m_{24} = I$ the first moment of inertia of mass, and $m_{44} = m_{55} = J$ the second (polar) moment of inertia of mass. In eqn (32), only the rigid-body displacements are considered for the kinetic energy to parallel with the formulation of eqn (14). Using eqns (9) and (26), the kinetic energy is rewritten in the form:

$$T_C = \tfrac{1}{2}\{\dot{\Delta}^R\}^T[M_C]\{\dot{\Delta}^R\}, \tag{34}$$

where $[M_C]$, the reduced mass matrix for a continuum plate, is now given by:

$$[M_C] = A_e[\bar{m}]_{eq}. \tag{35}$$

It is found that $[\bar{m}]_{eq}$ is almost identical to $[m]_{eq}$, except for the elements $\bar{m}_{44_{eq}}$ and $\bar{m}_{55_{eq}}$ which are related by:

$$\bar{m}_{44_{eq}} = m_{44_{eq}} + \frac{b^2}{12} m_{33_{eq}}, \tag{36}$$

$$\bar{m}_{55_{eq}} = m_{55_{eq}} + \frac{a^2}{12} m_{33_{eq}}, \tag{37}$$

where a and b are the dimensions of finite-element in the x- and y-directions, respectively.

2.4. *Equivalent continuum structural properties*

As described in the procedure of continuum modeling, the structural equivalent properties of continuum plate can be obtained from the concept of energy equivalence: $V_L = V_C$ and $T_L = T_C$. Thus simply equating eqns (19) and (20) to eqns (31) and (35), respectively, one obtains:

$$[E]_{eq} = \frac{1}{A_e}[K_L], \tag{38}$$

$$[\bar{m}]_{eq} = \frac{1}{A_e}[M_L]. \tag{39}$$

Therefore the equivalent continuum plate rigidities $[E]_{eq}$ and inertias $[m]_{eq}$ for a lattice plate can be readily calculated from quite simplified formulas of eqns (36)–(39). The need is only to compute the reduced stiffness and mass matrices for a representative lattice cell by use of eqns (19) and (20).

3. NUMERICAL TESTS AND DISCUSSIONS

In order to evaluate the proposed continuum method by means of numerical tests, three types of repeating cells, as shown in Fig. 2, are considered. Among very small number of studies on the continuum modeling of lattice plates, Sun *et al.* (1988), Noor *et al.* (1978),

Table 2. Geometric and material properties of lattice elements

Geometric and material properties		Lattice plates Type-A	Type-B	Type-C
ρ (kg m^{-3})		2768	2768	2768
E (N m^{-2})		71.7×10^9	71.7×10^9	71.7×10^9
a, b, h (m)		7.5, 7.5, 7.5	10.6, 10.6, 7.5	7.5, 7.5, 7.5
L_x, L_y (m)		75, 75 (10 Bays)	60, 60 (8 Bays)	60, 60 (8 Bays)
A(m^2)	———	40×10^{-6}	40×10^{-6}	80×10^{-6}
	———	5×10^{-6}	50×10^{-6}	—
	═══	—	25×10^{-7}	40×10^{-6}
	-·-·-·-	—	10×10^{-6}	—
	=·=·=·=	—	80×10^{-6}	—

and Flower and Schmidt (1971) considered the same lattice plates as Type-A, Type-B and Type-C models, respectively.

Sun *et al.* (1988) developed the continuum model by relating the deformation characteristics of a repeating cell to those of a continuum element by use of static analysis. Noor *et al.* (1978) developed the continuum models by expanding the nodal displacement of a lattice member in Taylor's series, and by equating the strain and kinetic energies of the lattice and continuum plates. Finally, Flower and Schmidt (1971) used the discrete field approach to obtain the approximated differential equations from the governing difference equations of the lattice plate. In the present study, their numerical results are compared with the results obtained by the present continuum method. For easy comparison, the same geometric and material properties of lattice elements that they used in their works are also used in the present study. Details of geometric and material properties are listed in Table 2. Note that ρ is the mass density per unit volume, E Young's modulus, A a cross-sectional area of lattice element, and a, b, and h the dimensions of a representative lattice cell in the x-, y- and z-directions, respectively. L_x and L_y are the dimensions of lattice plate in the x- and y-directions. For the vibration analysis, each lattice model is also assumed to be subjected to the same boundary conditions as considered by Sun *et al.* (1988) and Noor *et al.* (1978). That is, Type-A is clamped along one edge and free on the other edges, and Type-B and Type-C are simply supported on all edges.

Table 3 compares the equivalent structural properties of three lattice plate models. The equivalent structural properties by the present method are found to be very close to the

Table 3. Equivalent continuum plate rigidities ($\times 10^5$) and inertias

Equivalent rigidities and inertias	Type-A Present	Sun (1988)	Type-B Present	Noor (1978)	Type-C Present	Power (1971)
$A_{11} = A_{22}$ (N m^{-1})	17.02	16.95	17.16	16.82	16.34	—‡
A_{66} (N m^{-1})	1.044	1.980	4.394	4.394	1.041	—
A_{12} (N m^{-1})	1.044	1.044	4.394	4.394	1.041	—
$A_{16} = A_{26}$ (N m^{-1})	0.0	0.0	0.0	—	0.0	—
$B_{11} = B_{22}$ (N)	0.0	0.0	14.56	14.56	0.0	—
$B_{12} = B_{66}$ (N)	0.0	0.0	3.802	3.802	0.0	—
$B_{16} = B_{26}$ (N)	0.0	0.0	0.0	—	0.0	—
$D_{11} = D_{22}$ (N m)	224.6	224.6	236.5	236.6	215.1	215.1
$D_{12} = D_{66}$ (N m)	9.506	9.500	61.79	61.79	0.0	—
$D_{16} = D_{26}$ (N m)	0.0	0.0	0.0	—	0.0	—
$S_{44} = S_{55}$ (N m^{-1})	1.044	1.044	0.338	0.338	0.416	—
S_{45} (N m^{-1})	0.0	0.0	0.0	—	0.0	—
F_{ij} (N m^{-1})†	0.0	0.0	0.0	—	0.0	—
H_{ij} (N)†	0.0	0.0	0.0	—	0.0	—
m (kg m^{-2})	0.221	0.221	0.178	0.178	0.263	0.263
I (kg m^{-1})	0.0	0.0	0.142	0.142	0.0	—
J (kg)	3.447	2.242	2.656	2.368	2.477	—
Other inertias	0.0	—	0.0	—	0.0	—

† $i = 1, 2, 6$; $j = 4, 5$.
‡ (—) indicates the property cannot be given by the corresponding theory.

Table 4. Natural frequencies (Hz) of lattice plates

Mode no.	Direct sol.	Type-A Continuum Sol. Present	Type-A Continuum Sol. Sun (1988)	Direct sol.	Type-B Continuum Sol. Present	Type-B Continuum Sol. Noor (1978)
1	0.905	0.918	0.920	4.505	4.509	4.481
2	1.090	1.134	1.142	7.731	7.692	7.669
3	2.007	2.046†	2.046†	10.082	9.910	9.891
4	4.001	4.128	4.157	11.220	11.190	11.161
5	4.234	4.350	4.399	12.840	13.096†	13.094†
6	4.859	5.257	5.322	13.062	12.814	12.793
7	5.924	6.084†	6.081†	14.655	14.791	14.726
8	6.477	6.892	6.999	15.580	15.160	15.146
9	8.579	7.952	8.014	16.178	16.049	16.001
10	8.726	8.937	9.163	17.815	17.961	18.319

† Inplane modes.

results by other methods, especially to the results by Noor *et al.* (1978). The influence of the geometric and material properties of lattice elements on the equivalent structural properties can be observed from Table 3 (see the axial-bending coupling rigidities B_{ij}). The advantage of the present continuum method is that it can give us all kinds of equivalent structural properties, while the other methods give only limited number of structural properties as shown in Table 3. To evaluate the accuracy and validity of the present continuum method, the natural frequencies by the present continuum method are also compared with those by other methods, i.e. the direct solutions by the conventional full-scale finite-element analysis of the original lattice plates (Sun *et al.*, 1988, and Noor *et al.*, 1978), and the continuum solutions by the continuum methods of Sun *et al.* (1988), Noor *et al.* (1978) and Power and Schmidt (1971). The equivalent structural properties of Table 3 are used in calculating the continuum solutions which are given in Table 4.

For the free vibration analyses of continuum plates, nine-noded rectangularLagrange element is used. For the sufficient convergence of the 10 lowest fundamental frequencies, the analysis was repeated with increasing the number of finite-elements. For the natural frequencies shown in Table 4, and for the normal mode shapes shown in Figs 6 and 7, 8 × 8 meshes of elements are used along with the reduced Gaussian integration rule, which is usually adopted to overcome overstiff solutions. And the shear correction factor 5/6 is considered in the expression for the shear rigidity [S] to allow for warping of the plate cross-section (Hinton and Bícaníc, 1979). Table 4 shows the accuracy of the natural frequencies with respect to the direct solutions. It is found that the present continuum method gives improved results, which are rather close to the direct solutions by the conventional finite-element analysis than to the solutions by other methods in general. Many authors (see Noor *et al.*, 1978 and Sun *et al.*, 1988) have demonstrated that continuum

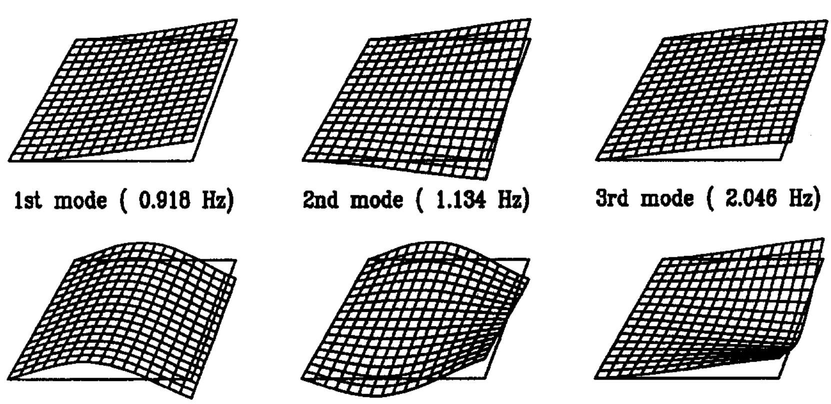

Fig. 6. Lowest six normal modes for the lattice plate Type-A clamped along one edge (third mode is the inplane mode).

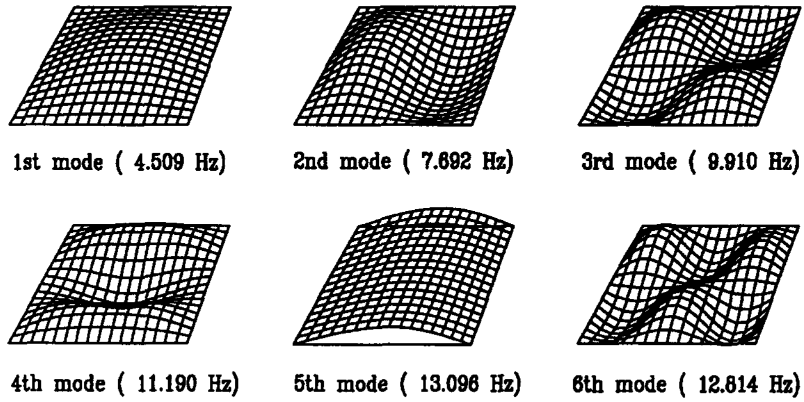

Fig. 7. Lowest six normal modes for the simply supported lattice plate Type-B (fifth mode is the inplane mode).

methods are highly accurate especially when the wavelength of the vibration mode encompasses more than one repeating cell. Thus, the continuum method developed in this paper, as a new member of continuum methods, is also expected to provide satisfactory continuum models for the lower modes vibrations of large flexible platelike lattice structures in space.

4. CONCLUSION

Based on the concept of energy equivalence, a new continuum modeling method is introduced for the large platelike periodic lattice plates. The key to continuum modeling involves the introduction of appropriate continuum DOFs and the use of existing well-defined finite-element matrices for easy calculation of strain and kinetic energies contained in a representative lattice cell, and accordingly for easy derivation of equivalent structural properties of continuum plates. Numerical tests prove that the present continuum method gives very reliable and competitive equivalent structural properties compared to the other existing continuum methods.

Acknowledgements—The author is pleased to acknowledge partial supports from the Korea Science and Engineering Foundation under Grants KOSEF 901-0910-033-2 and the Korea Research Foundation under NON Directed Research Fund.

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